Quantum Field Theory

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Chapter 1

Appendix A: Reference Formulae

1.1 Scalar Field

1.1.1 Real Scalar Field

Introduction

Consider the Lagrangian

$$\mathcal{L} = \frac{1}{2}(\partial\phi)^2 - \frac{1}{2}m^2\phi^2$$

for the free real scalar field in Shroedinger representation

$$\phi = \phi_S(\vec{x})$$

and the time-independent potential

$$V(\phi) = \frac{1}{2}m^2\phi^2$$

The equations of motion (Euler-Lagrange equations) related to this Lagrangian are the Klein-Gordon equations

$$\partial_\mu\partial^\mu\phi + m\phi^2 = 0$$

Quantization

We impose the canonical quantization

$$[\phi(\vec{x}), \pi(\vec{y})] = i\delta(\vec{x} - \vec{y}) \quad [\phi(\vec{x}), \phi(\vec{y})] = 0 \quad [\pi(\vec{x}), \pi(\vec{y})] = 0$$

Define the ladder operators such that

$$\phi(\vec{x}) := \int \frac{d\vec{p}}{(2\pi)^3} \frac{1}{\sqrt{2\omega_{\vec{p}}}} \left(a_{\vec{p}} e^{i\vec{p}\cdot\vec{x}} + a^\dagger_{\vec{p}} e^{-i\vec{p}\cdot\vec{x}} \right)$$

$$\pi(\vec{x}) := -i \int \frac{d\vec{p}}{(2\pi)^3} \frac{\sqrt{\omega_{\vec{p}}}}{\sqrt{2}} \left(a_{\vec{p}} e^{i\vec{p}\cdot\vec{x}} - a^\dagger_{\vec{p}} e^{-i\vec{p}\cdot\vec{x}} \right)$$

with

$$\omega_{\vec{p}} = \sqrt{\vec{p}^2 + m^2}$$

We get the following commutation relations for the ladder operators

$$[a_{\vec{p}}, a^\dagger_{\vec{p}'}] = (2\pi)^3 \delta(\vec{p} - \vec{p}') \quad [a_{\vec{p}}, a_{\vec{p}'}] = 0 \quad [a^\dagger_{\vec{p}}, a^\dagger_{\vec{p}'}] = 0$$

Operators

We get the following Hamiltonian

$$\mathcal{H} = \frac{1}{2} \int \frac{d\vec{p}}{(2\pi)^3} \omega_{\vec{p}} \left(2a^\dagger_{\vec{p}} a_{\vec{p}} + (2\pi)^3 \delta(0) \right)$$

With eigenvalues equation

$$\mathcal{H} |\vec{p}\rangle = \omega_{\vec{p}} |\vec{p}\rangle$$

The momentum operator

$$\vec{\mathcal{P}} = \frac{1}{2} \int \frac{d\vec{p}}{(2\pi)^3} \ \vec{p} \ a^\dagger_{\vec{p}} a_{\vec{p}}$$

with eigenvalues equation

$$\vec{\mathcal{P}} |\vec{p}\rangle = \vec{p} |\vec{p}\rangle$$

We have the four vector

$$p^\mu = \left(\mathcal{H}, \vec{\mathcal{P}} \right)$$

Fock's space: set of all the states such that

$$|\vec{p_1} \dots \vec{p_n}\rangle = \sqrt{2\omega_{\vec{p_1}}} \dots \sqrt{2\omega_{\vec{p_n}}} a^\dagger_{\vec{p_1}} \dots a^\dagger_{\vec{p_n}} |0\rangle$$

Hence

$$\langle \vec{p}' | \vec{p} \rangle = 2\omega_{\vec{p}} (2\pi)^3 \delta(\vec{p}' - \vec{p})$$

$$\phi(\vec{x})|0\rangle = \int \frac{d\vec{p}}{(2\pi)^3} \frac{1}{\sqrt{2\omega_{\vec{p}}}} e^{-i\vec{p}\cdot\vec{x}} |\vec{p}\rangle$$

$$\langle \vec{p} | \phi(\vec{x}) | 0 \rangle = \sqrt{2\omega_{\vec{p}}} \ e^{-i\vec{p}\cdot\vec{x}}$$

1.1.2 Complex Scalar Field

Consider the Lagrangian $\mathcal{L}$

$$\mathcal{L} = \partial_\mu \phi \, \partial^\mu \phi^* - m^2 \phi \phi^*$$

This is a theory of two uncoupled free real scalar fields, since splitting the free complex scalar field ϕ into its real and imaginary part as follows

$$\phi = \frac{\phi_1 + i\phi_2}{\sqrt{2}}$$

we obtain

$$\mathcal{L} = \mathcal{L}_{real}(\phi_1) + \mathcal{L}_{real}(\phi_2)$$

Chapter 2

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2.1 Text

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