

Quantum Field Theory



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Chapter 1

Appendix A: Reference Formulae

1.1 Scalar Field

1.1.1 Real Scalar Field

Introduction

Consider the Lagrangian

$$\mathcal{L} = \frac{1}{2}(\partial\phi)^2 - \frac{1}{2}m^2\phi^2$$

for the free real scalar field in Shroedinger representation

$$\phi = \phi_S(\vec{x})$$

and the time-independent potential

$$V(\phi) = \frac{1}{2}m^2\phi^2$$

The equations of motion (Euler-Lagrange equations) related to this Lagrangian are the Klein-Gordon equations

$$\partial_\mu\partial^\mu\phi + m\phi = 0$$

Quantization

We impose the canonical quantization

$$[\phi(\vec{x}), \pi(\vec{y})] = i\delta(\vec{x} - \vec{y}) \quad [\phi(\vec{x}), \phi(\vec{y})] = 0 \quad [\pi(\vec{x}), \pi(\vec{y})] = 0$$

Define the ladder operators such that



$$\begin{aligned}\phi(\vec{x}) &:= \int \frac{d\vec{p}}{(2\pi)^3} \frac{1}{\sqrt{2\omega_{\vec{p}}}} \left(a_{\vec{p}} e^{i\vec{p}\cdot\vec{x}} + a_{\vec{p}}^\dagger e^{-i\vec{p}\cdot\vec{x}} \right) \\ \pi(\vec{x}) &:= -i \int \frac{d\vec{p}}{(2\pi)^3} \frac{\sqrt{\omega_{\vec{p}}}}{\sqrt{2}} \left(a_{\vec{p}} e^{i\vec{p}\cdot\vec{x}} - a_{\vec{p}}^\dagger e^{-i\vec{p}\cdot\vec{x}} \right)\end{aligned}$$

with

$$\omega_{\vec{p}} = \sqrt{\vec{p}^2 + m^2}$$

We get the following commutation relations for the ladder operators

$$[a_{\vec{p}}, a_{\vec{p}'}^\dagger] = (2\pi)^3 \delta(\vec{p} - \vec{p}') \quad [a_{\vec{p}}, a_{\vec{p}'}] = 0 \quad [a_{\vec{p}}^\dagger, a_{\vec{p}'}^\dagger] = 0$$

Operators

We get the following Hamiltonian

$$\mathcal{H} = \frac{1}{2} \int \frac{d\vec{p}}{(2\pi)^3} \omega_{\vec{p}} \left(2a_{\vec{p}}^\dagger a_{\vec{p}} + (2\pi)^3 \delta(0) \right)$$

With eigenvalues equation

$$\mathcal{H}|\vec{p}\rangle = \omega_{\vec{p}}|\vec{p}\rangle$$

The momentum operator

$$\vec{\mathcal{P}} = \frac{1}{2} \int \frac{d\vec{p}}{(2\pi)^3} \vec{p} a_{\vec{p}}^\dagger a_{\vec{p}}$$

with eigenvalues equation

$$\vec{\mathcal{P}}|\vec{p}\rangle = \vec{p}|\vec{p}\rangle$$

We have the four vector

$$p^\mu = \left(\mathcal{H}, \vec{\mathcal{P}} \right)$$

Fock's space: set of all the states such that

$$|\vec{p}_1 \dots \vec{p}_n\rangle = \sqrt{2\omega_{\vec{p}_1}} \dots \sqrt{2\omega_{\vec{p}_n}} a_{\vec{p}_1}^\dagger \dots a_{\vec{p}_n}^\dagger |0\rangle$$

Hence

$$\begin{aligned}\langle \vec{p}' | \vec{p} \rangle &= 2\omega_{\vec{p}} (2\pi)^3 \delta(\vec{p}' - \vec{p}) \\ \phi(\vec{x})|0\rangle &= \int \frac{d\vec{p}}{(2\pi)^3} \frac{1}{\sqrt{2\omega_{\vec{p}}}} e^{-i\vec{p}\cdot\vec{x}} |\vec{p}\rangle \\ \langle \vec{p}' | \phi(\vec{x}) | 0 \rangle &= \sqrt{2\omega_{\vec{p}'}} e^{-i\vec{p}'\cdot\vec{x}}\end{aligned}$$



1.1.2 Complex Scalar Field

Consider the Lagrangian \mathcal{L}

$$\mathcal{L} = \partial_\mu \phi \partial^\mu \phi^* - m^2 \phi \phi^*$$

This is a theory of two uncoupled free real scalar fields, since splitting the free complex scalar field ϕ into its real and imaginary part as follows

$$\phi = \frac{\phi_1 + i\phi_2}{\sqrt{2}}$$

we obtain

$$\mathcal{L} = \mathcal{L}_{real}(\phi_1) + \mathcal{L}_{real}(\phi_2)$$



Chapter 2

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2.1 Text

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